Algebra & Calculus

Limits

Zeno's Limit


MY CLIENT COULDN'T HAVE KILLED ANYONE WITH THIS ARROW, AND I CAN *PROVE* IT!
I'D LIKE TO EXAMINE YOUR PROOF, ZENO. YOU MAY APPROACH THE BENCH.
—BUT NEVER REACH IT!

Is 2 euro the best possible return If I invest €1 after 1 year with an interest rate of 100%?

How often interest is compounded	Final Value F	
Yearly	$F = 1(1+1)^{1}$	$= 2.0$
Every 6 months	$F = 1\left(1+\frac{1}{2}\right)^{2}$	$= 2.25$
Every 3 months	$F = 1\left(1+\frac{1}{4}\right)^{4}$	$= 2.44140625$
Every month	$F = 1\left(1+\frac{1}{12}\right)^{12}$	$= 2.61303529$
Every week	$F = 1\left(1+\frac{1}{52}\right)^{52}$	$= 2.69259695$
Every day	$F = 1\left(1+\frac{1}{365}\right)^{365}$	$= 2.71456748$
Every hour	$F = 1\left(1+\frac{1}{8760}\right)^{8760}$	$= 2.71812669$
Every minute	$F = 1\left(1+\frac{1}{525600}\right)^{525600}$	$= 2.71827923$
Every second	$F = 1\left(1+\frac{1}{31536000}\right)^{31536000}$	$= 2.71828162$

2.7

2.71

2.718

2.7182

2.71828

2.718281

2.7182818

2.71828182

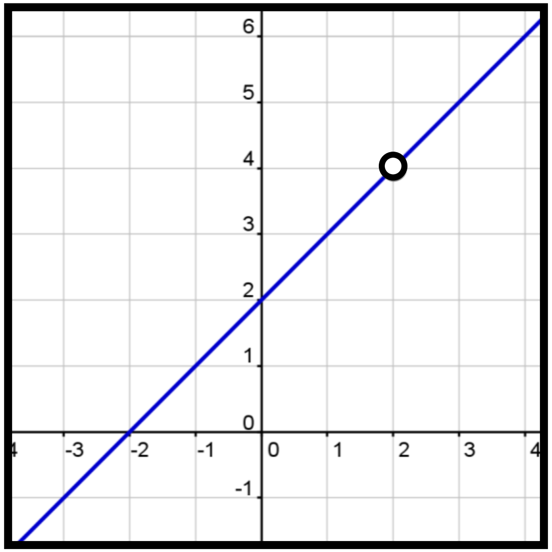
2.718281828

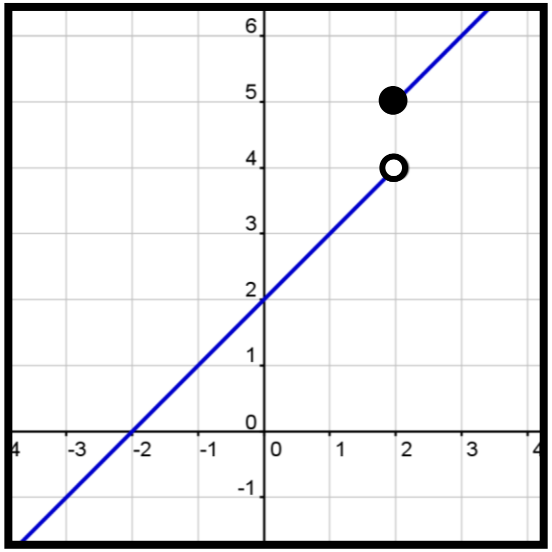
2.7182818284.............

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n} = e$$

Practical Purposes

Can we say y practically equals 4 when x = 2 in the following?





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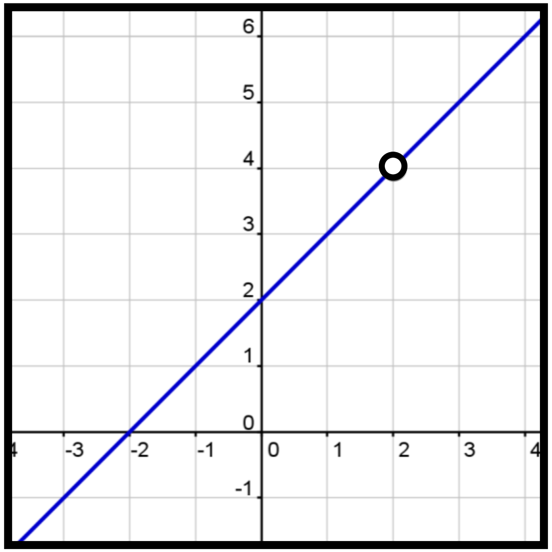

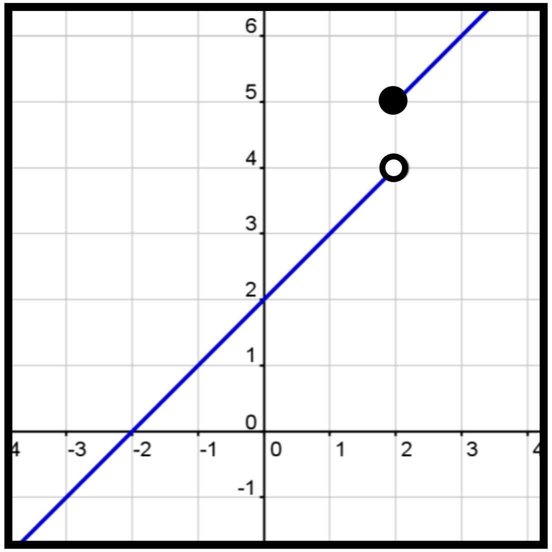

When x = 2:
y is approaching 4 from the left and the right

When x =2:
y is approaching 4 from the left and 5 from the right

The practical value we use is called a limit and exists if the function approaches the same value from the left and right.

Assuming that the domains of the following functions are over R, write out the accurate domain for each function.

$$f(x) = x + 3$$

$$g(x) = \frac{x^2 - 9}{x - 3}$$

$$h(x) = \frac{1}{x - 3}$$

Assuming that the domains of the following functions are over R, write out the accurate domain for each function.

$$f(x) = x + 3 \qquad x \in R$$

$$g(x) = \frac{x^2 - 9}{x - 3} \qquad x \in R,\ \ x \neq 3$$

$$h(x) = \frac{1}{x - 3} \qquad x \in R, x \neq 3$$

Concept of a limit
Student Activity

X	2.9	2.99	2.999	2.9999		3		3.0001	3.001	3.01	3.1
$f(x) = x + 3$											
$g(x) = \frac{x^2 - 9}{x - 3}$					…		…				
$h(x) = \frac{1}{x - 3}$											

- When x approaches 3 (but $x \neq 3$) does the value of each of the following functions approach a fixed value?
- If it does find the value

Given: $f(x) = x + 3$ *Ans:*______

$g(x) = \frac{x^2-9}{x-3}$ *Ans:*______

$h(x) = \frac{1}{x-3}$ *Ans:*______

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$f(x) = x + 3$	5.9	5.99	5.999	5.9999		6		6.0001	6.001	6.01	6.1
$g(x) = \frac{x^2 - 9}{x - 3}$	5.9	5.99	5.999	5.9999	…	?	…	6.0001	6.001	6.01	6.1
$h(x) = \frac{1}{x - 3}$	-10	-100	-1000	-10000		?		10000	1000	100	10

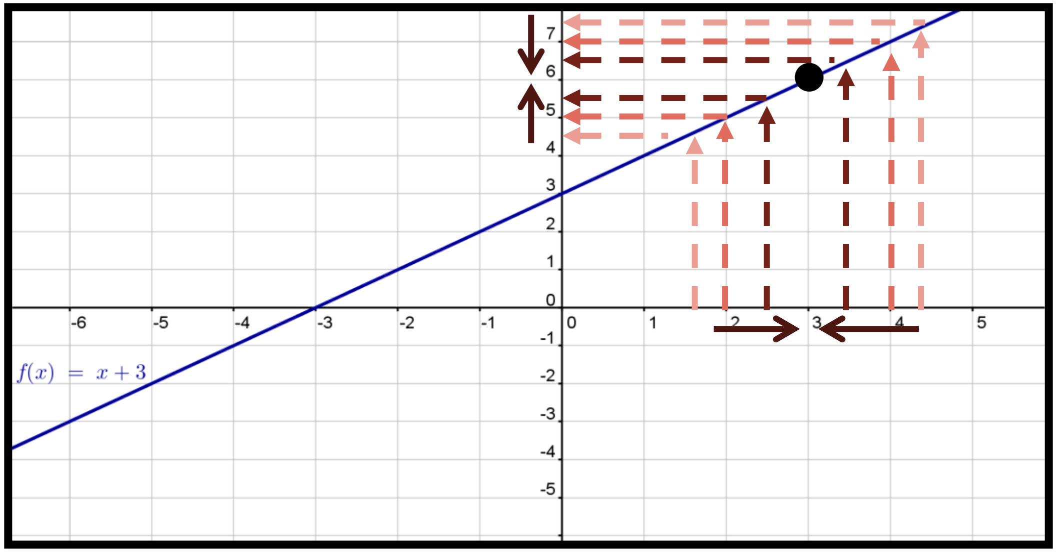
- When x approaches 3 (but $x \neq 3$) does the value of each of the following functions approach a fixed value?
- If it does find the value

Given: $f(x) = x + 3$ $Ans:$ 6_____

$g(x) = \frac{x^2-9}{x-3}$ $Ans:$ 6_____

$h(x) = \frac{1}{x-3}$ $Ans:$ No____

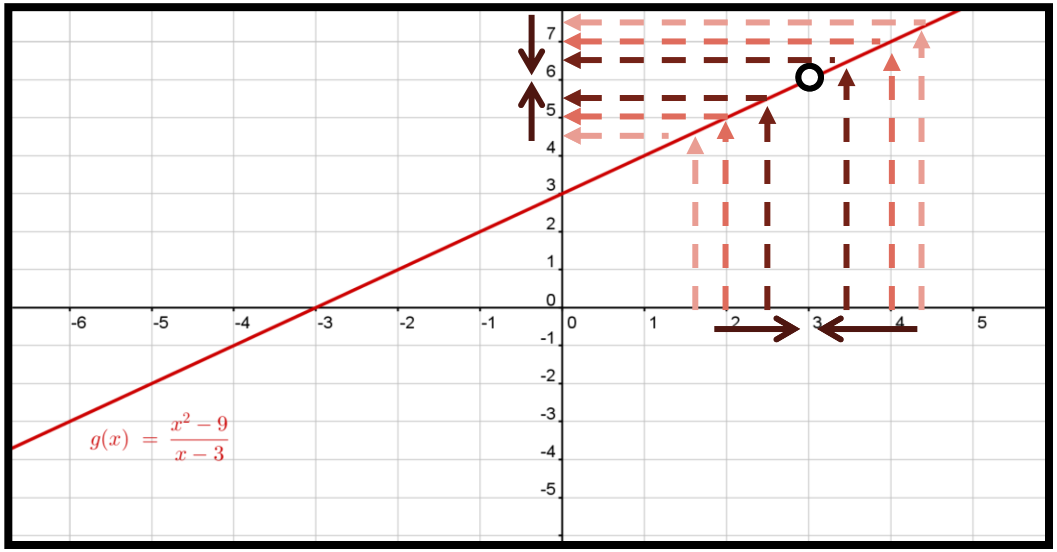
Understanding the Geometrical Meaning of 'Limits of Functions'



$$f(x) = x + 3$$

$$\lim_{x \to 3} f(x) = 6$$

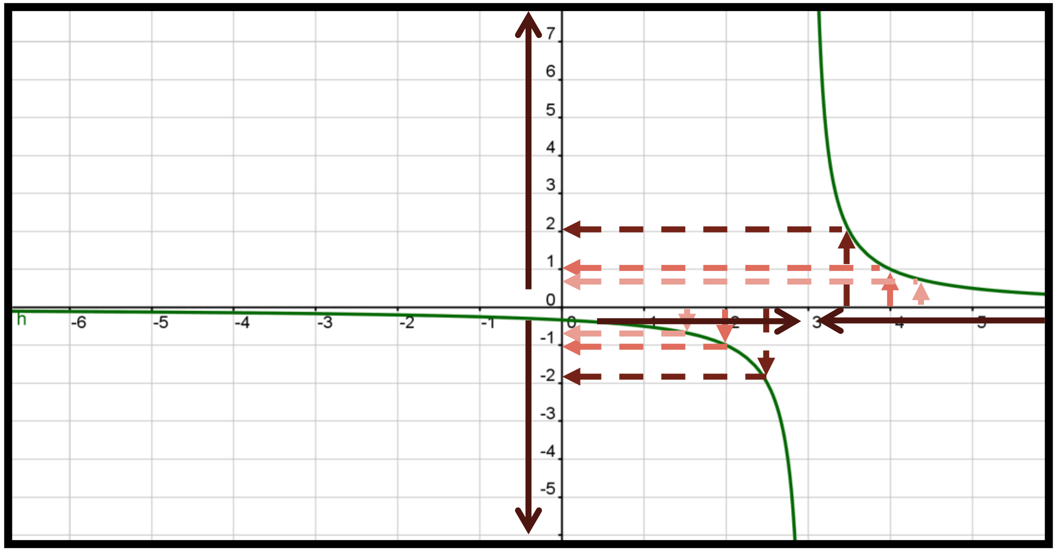
Understanding the Geometrical Meaning of 'Limits of Functions'



$$g(x) = \frac{x^2 - 9}{x - 3}$$

$$\lim_{x \to 3} g(x) = 6$$

Understanding the Geometrical Meaning of 'Limits of Functions'



$$h(x) = \frac{1}{x-3}$$

$$\lim_{x \to 3} h(x) \quad does\ not\ exist$$

Notation

The value of **<u>some</u>** functions approach a fixed value when x approaches 3.

$f(x) = x + 3$, the function approaches 6 when x approaches 3 (but $x \neq 3$).

- Notation: In maths, we denote "x approaches 3" as "$x \to 3$"
- We say : The limit of $f(x) = x + 3$ as x approaches 3 is 6
- Expressed as: $\lim\limits_{x \to 3} f(x) = 6$ or $\lim\limits_{x \to 3} (x + 3) = 6$

Similarly

The limit of $g(x) = \frac{x^2-9}{x-3}$ as x approaches 3 is 6,

can be expressed as

$$\lim_{x \to 3} g(x) = 6 \qquad \text{or} \qquad \lim_{x \to 3} \frac{x^2-9}{x-3} = 6$$

Where a limit does not exist

What about when $x \to 3$, $h(x) = \frac{1}{x-3}$?

This does not approach any fixed value.

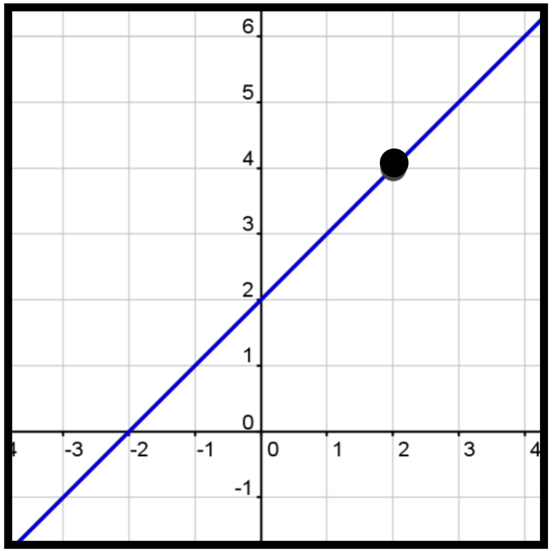
- We say:

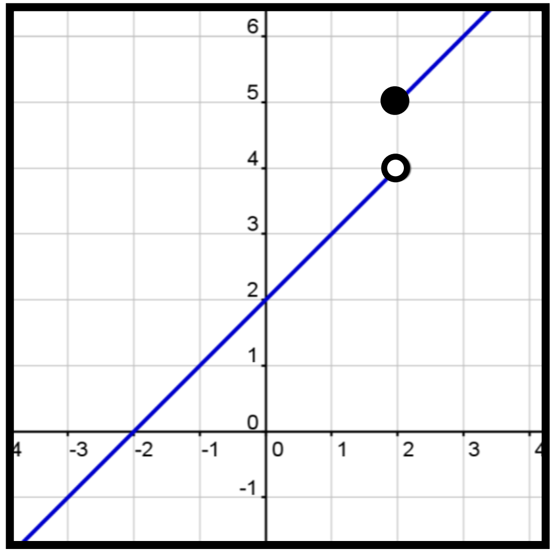
" The limit of $h(x) = \frac{1}{x-3}$ as x approaches 3 does not exist ",

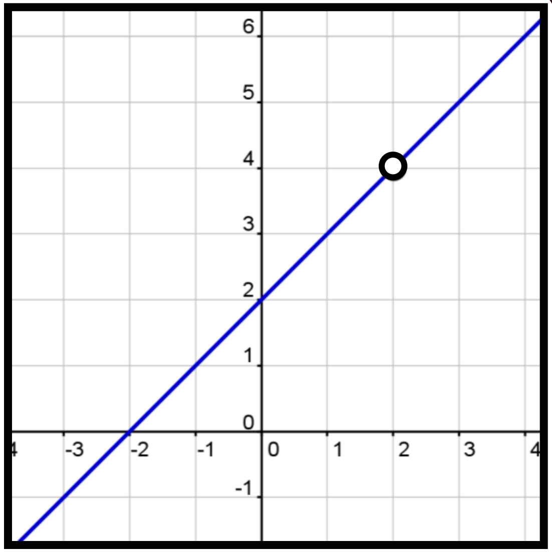
- can be expressed as :

"$\lim_{x \to 3} h(x)$ does not exist" or " $\lim_{x \to 3} \frac{1}{x-3}$ does not exist "

Continuous Functions







For a function to be continuous at a point it must fulfil all these 3 conditions:

1 The function must have a value at x_0.

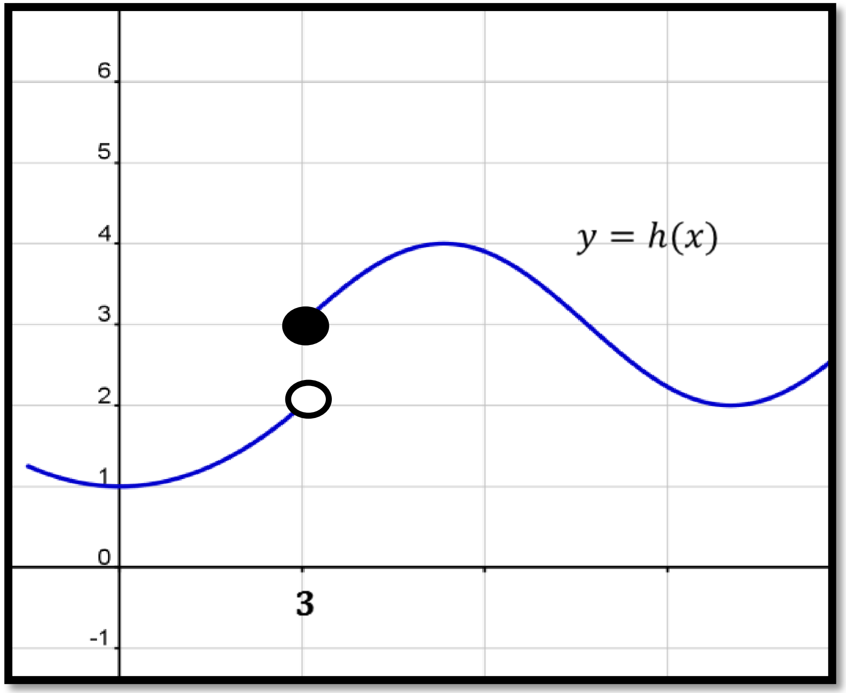
2 The function must have a limit at x_0.

3 Value = Limit.

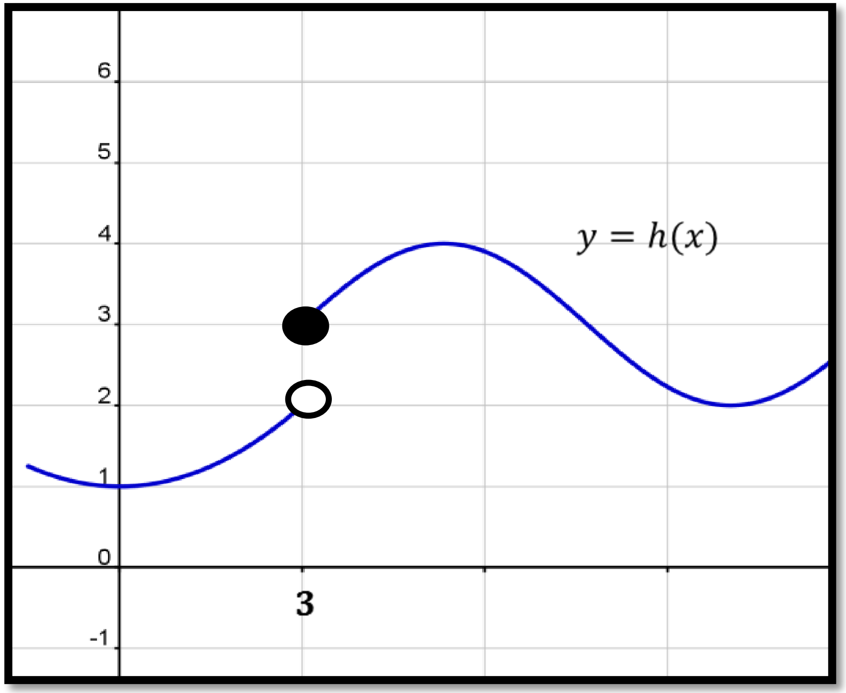
→ **Continuous.**

Are the above graphs continuous functions at x = 2?

Continuous – Yes / No



Continuous – Yes / No



$$y = h(x)$$

Is the function continuous at $x = 3$?

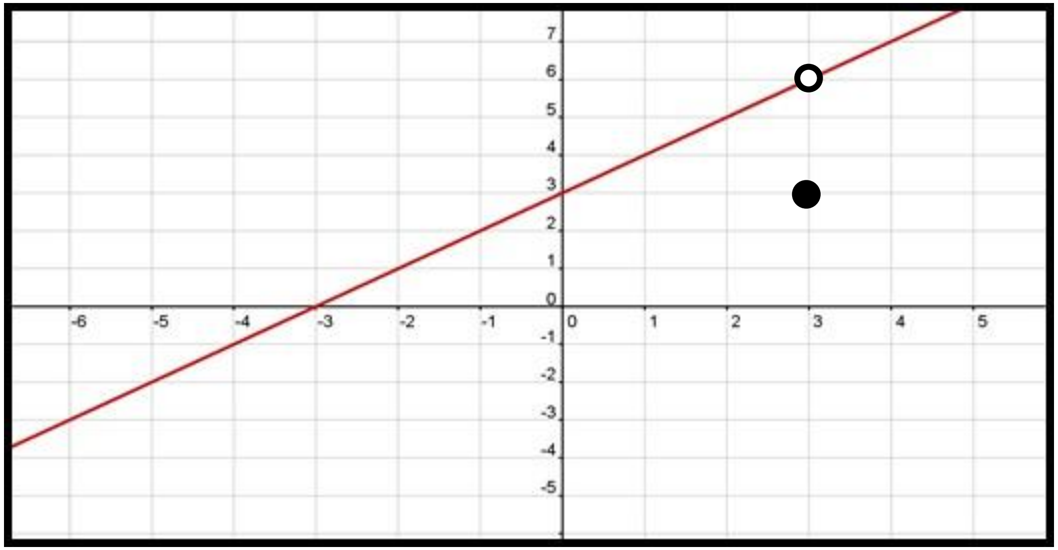
1 The function has a value at $x = 3$, i.e. $h(3) = 3$.

2 The function does not have a limit as $x \to 3$.

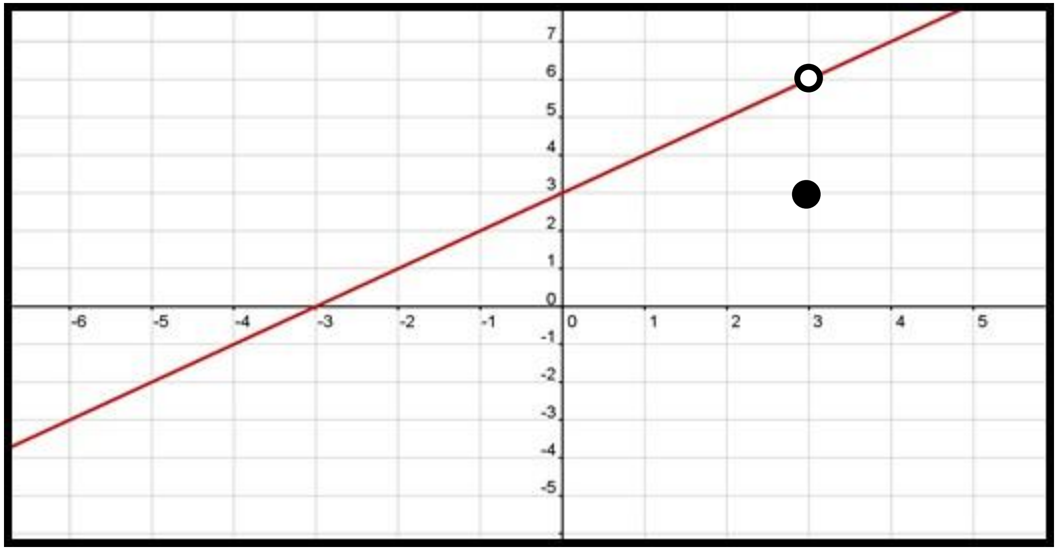
3 Value ≠ Limit.

→ **Not Continuous.**

Continuous – Yes / No



Continuous – Yes / No



$$y = k(x)$$

Is the function continuous at $x = 3$?

1 The function has a value at $x = 3$, i.e. $k(3) = 3$.

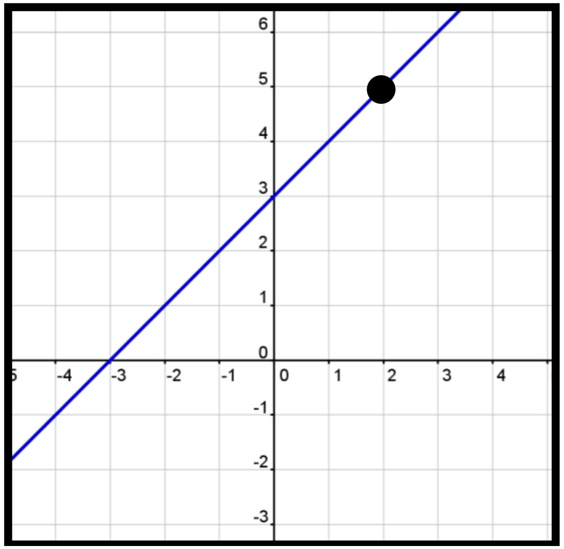
2 The function has a limit at $x = 3$, i.e. $\lim_{x \to 3} k(x) = 6$.

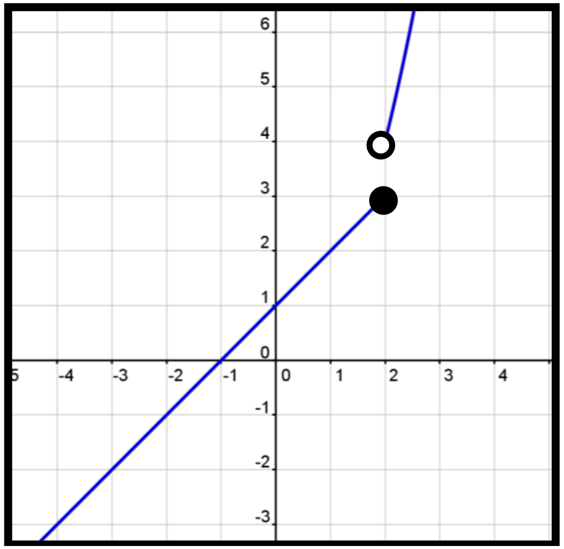
3 Value ≠ Limit.

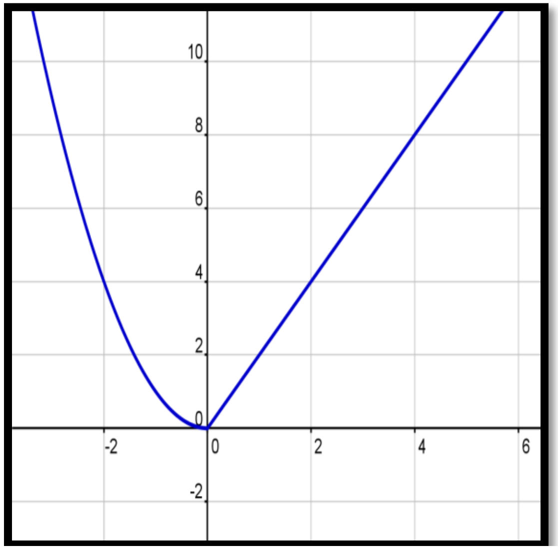
→ **Not Continuous.**

Don’t forget skills to reinforce

- Determine whether each of the following functions are continuous.
- If not state where the function is discontinuous

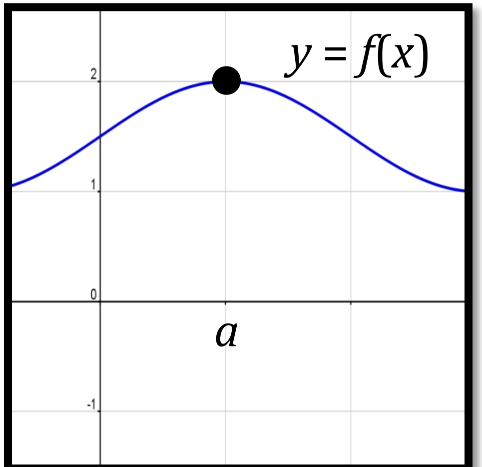




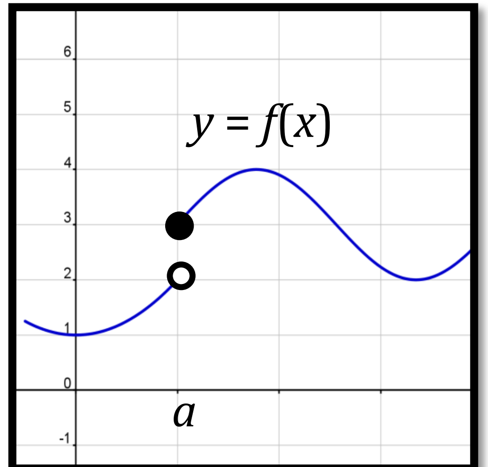


According to each of the following graphs of functions f(x):
(i) Find $\lim f(x)$ (ii) Determine whether f(x) is continuous at x =a

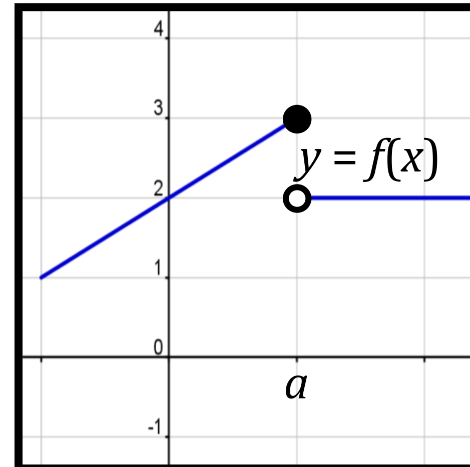
a)



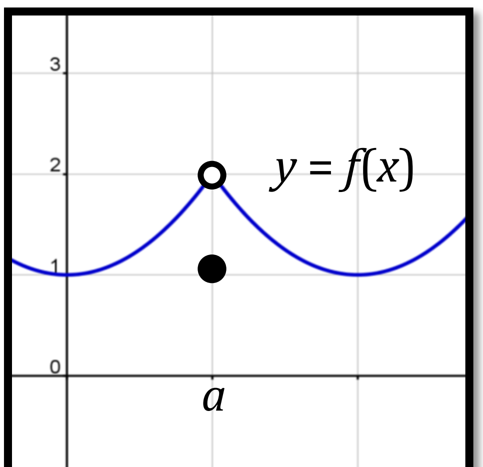
b)



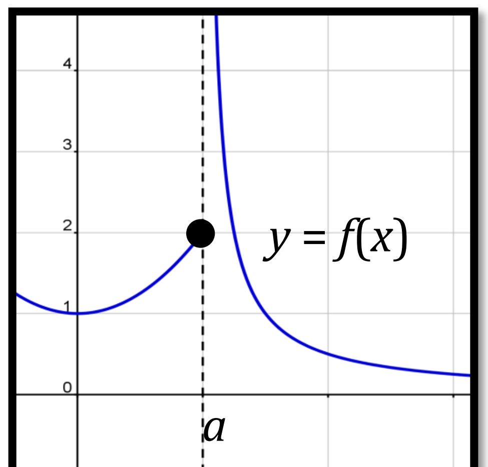
c)



d)



e)



f)